The diffusive flow of gases between two parallel plates

G. T. ROBERTS†

Department of Physics, University College of Wales, Aberystwyth, Wales

MS. received 19th August 1970, *in revised form 23rd November* 1970

Abstract. The flow rates of hydrogen, helium, argon, nitrogen, carbon dioxide, krypton and water vapour through both glass and silver parallel-plate channels have been measured for values of the ratio of the plate separation to the mean free path between 0·02 and 0·5. An equation for the diffusive flow is derived which takes account of the mode of scattering at the channel surface, which when compared with the experimental results shows that the scattering off the silver surfaces is diffusive, whereas that off the glass surfaces is mainly diffusive but with some specular reflection.

1. Introduction

A mode of flow which depends on the diffusion of individual molecules is called diffusive flow. The flow is usually supported by both intermolecular and molecular–surface collisions; however, in the limit when gas molecules rarely collide with one another in comparison with the number of collisions with the channel surface, the flow is called molecular flow.

An equation for the diffusive flow of gases through channels of circular cross section was derived by Present and Pollard (1948) by using the directional distribution method due to Knudsen (1909), but by assuming that collision partners move isotropically after an intermolecular collision. Hiby and Pahl (1952) reconsidered the work of Present and Pollard and derived an additional flow term to account for the nonisotropic distribution of collision partners, for both parallel-plate channels and channels of circular cross section. But unfortunately their equations are only applicable for values of α (the ratio of the channel width h to the mean free path λ) less than approximately 0·1, hence their equations could not be directly incorporated in general flow equations, such as those described by Weber (1954), Fryer (1966) and Roberts (1969).

All the equations discussed were derived by assuming diffusive reflection of molecules off the channel surfaces, with any deviation from the cosine law being accounted for by the term $(2-f)/f$ derived by von Smoluchowski (1910), where f is the momentum accommodation coefficient. Hence no account was taken of the possibility that the mode of molecular reflection off solid surfaces depends on the incident direction of the molecules. In the present paper the equations of Hiby and Pahl are amended to account for the angular distribution of specular reflection and to cover all values of α.

The previous paper by the author, Roberts (1969), included a derivation of the viscous and slip flow terms for all values of α, hence by adding these two terms to the diffusive term derived in this paper a general flow equation results which is applicable for all values of α.

The experimental data on diffusive flow between parallel plates have been limited to the results of Gaede (1913), Dong (1956) and Roberts (1969). In all the experiments the conditions were such that the differential pressure across the channels

† Now at the Department of Physics, Glamorgan Polytechnic, Llantwit Road, Treforest, Pontypridd.

was greater than or approximately equal to the mean pressure in the channel, therefore the flow over a small range of α could not be studied independently of the flows at neighbouring values of α.

This present paper describes a series of experiments carried out on the diffusive flow of hydrogen, helium, nitrogen, argon, carbon dioxide, krypton and water vapour through parallel-plate glass and silver channels of separation between $1 \cdot 6 \, \mu m$ and $14 \cdot 5 \, \mu m$ and for values of α between $0 \cdot 02$ and $0 \cdot 5$.

The results are interpreted in the light of the theoretical equations which are derived to take account of the angular distribution of specular reflection off the channel surfaces.

2. Theory

2.1. *Isotropic scattering after both intermolecular and molecular–surface collisions*

To a first approximation Hiby and Pahl (1952) assumed isotropic molecular distribution after an intermolecular collision and showed that the flow equation for a parallel-plate channel can be derived by using the method developed by Present and Pollard (1948) for long channels of circular cross section. Let figure 1 represent an
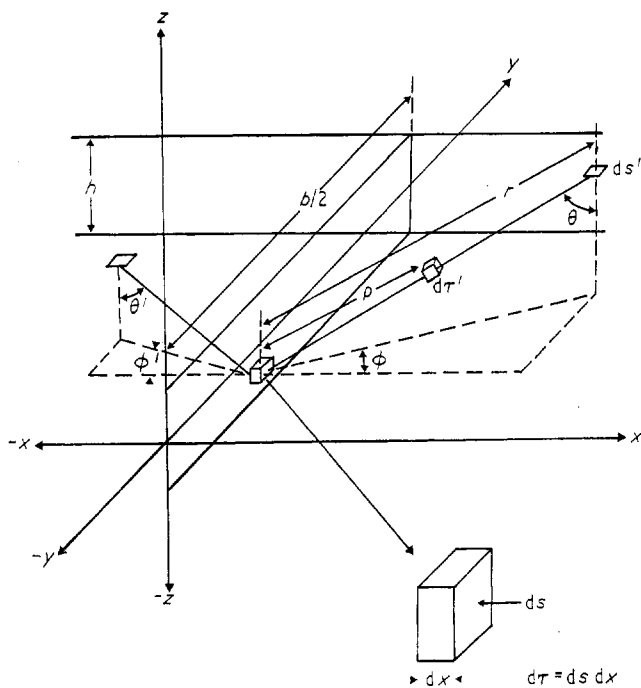

Figure 1. Cross section of the channel.

element of channel length and consider the net transport of molecules across the element of area ds. The molecules crossing ds will either arrive directly from the wall or be deflected through ds by an intermolecular collision at $d\tau'$. Therefore two groups of molecules will be considered, namely dN_w and dN_g respectively. For a

parallel-plate channel Hiby and Pahl showed that

$$-\,\mathrm{d}N_\mathrm{w} = \frac{n\bar{c}}{4\pi}\,\mathrm{d}s\,\cos\phi\,\sin^2\theta\,\exp\left(-\frac{r}{\lambda}\right)\,\mathrm{d}\theta\,\mathrm{d}\phi \tag{1}$$

and

$$-\,\mathrm{d}N_\mathrm{g} = \frac{n\bar{c}}{4\pi\lambda}\,\mathrm{d}s\,\cos\phi\,\sin^2\theta\,\exp\left(-\frac{\rho}{\lambda}\right)\,\mathrm{d}\theta\,\mathrm{d}\phi\,\mathrm{d}\rho \tag{2}$$

where n is the molecular number density, $\bar{c}$ is the molecular mean velocity, and parameters r, ϕ, θ and ρ are as shown in figure 1. After substituting for

$$n = n_0 + x\,\frac{\mathrm{d}n}{\mathrm{d}x}$$

where

$$x_r = \left(\frac{h}{2}-z\right)\cos\phi\,\sin\theta \qquad \text{and} \qquad x_\rho = \rho\cos\phi\,\sin\theta$$

and integrating, the two equations yield

$$N_\mathrm{w} = -\,\bar{c}bh^2\,\frac{\mathrm{d}n}{\mathrm{d}x}\int_0^{\pi/2}\left\{\frac{\cos\theta}{2\alpha^2}-\left(\frac{1}{2\alpha}+\frac{\cos\theta}{2\alpha^2}\right)\exp\left(-\frac{\alpha}{\cos\theta}\right)\right\}\sin^3\theta\,\mathrm{d}\theta \tag{3}$$

and

$$N_\mathrm{g} = -\,\bar{c}bh^2\,\frac{\mathrm{d}n}{\mathrm{d}x}\int_0^{\pi/2}\left\{\frac{1}{2\alpha}-\frac{\cos\theta}{\alpha^2}+\left(\frac{\cos\theta}{\alpha^2}+\frac{1}{2\alpha}\right)\exp\left(-\frac{\alpha}{\cos\theta}\right)\right\}\sin^3\theta\,\mathrm{d}\theta. \tag{4}$$

Present and Pollard in their work on channels of circular cross section considered the effect of variations in λ due to the total pressure gradient and showed that the sum of $N_\mathrm{w}+N_\mathrm{g}$ is unchanged although the individual components are different. This is also true for a parallel-plate channel. By using the procedure described in Appendix 3 of their paper, equations (1) and (2) become:

$$-\,\mathrm{d}N_\mathrm{w} = \frac{\bar{c}\,\mathrm{d}s}{4\pi}\,\frac{\mathrm{d}n}{\mathrm{d}x}\cos^2\phi\,\sin^3\theta\,\left(r-\frac{r^2}{2\lambda}\right)\exp\left(-\frac{r}{\lambda}\right)\,\mathrm{d}\theta\,\mathrm{d}\phi \tag{5}$$

and

$$-\,\mathrm{d}N_\mathrm{g} = \frac{\bar{c}\,\mathrm{d}s}{4\pi}\,\frac{\mathrm{d}n}{\mathrm{d}x}\cos^2\phi\,\sin^2\theta\,\left(\frac{2\rho}{\lambda}-\frac{\rho^2}{2\lambda^2}\right)\exp\left(-\frac{\rho}{\lambda}\right)\,\mathrm{d}\theta\,\mathrm{d}\phi\,\mathrm{d}\rho \tag{6}$$

which after integrating yields

$$N_\mathrm{w} = -\,\frac{\pi\bar{c}bh^2}{8}\,\frac{\mathrm{d}n}{\mathrm{d}x}\,f_\mathrm{w}(\alpha)$$

and similarly for N_g where

$$f_\mathrm{w}(\alpha) = \frac{2}{\pi}\int_0^{\pi/2}\frac{\sin^3\theta}{\cos\theta}\,\exp\left(-\frac{\alpha}{\cos\theta}\right)\,\mathrm{d}\theta \tag{7}$$

and

$$f_\mathrm{g} = \frac{4}{\pi}\int_0^{\pi/2}\left\{\frac{1}{\alpha}-\frac{\cos\theta}{\alpha^2}+\left(\frac{\cos\theta}{\alpha^2}-\frac{1}{2\cos\theta}\right)\exp\left(-\frac{\alpha}{\cos\theta}\right)\right\}\sin^3\theta\,\mathrm{d}\theta. \tag{8}$$

The sum of N_w and N_g from both equations (3) and (4) and equations (7) and (8) is given by

$$N = -\bar{c}bh^2 \frac{\mathrm{d}n}{\mathrm{d}x} \left\{ \frac{1}{3\alpha} - \frac{1}{8\alpha^2} + \int_0^{\pi/2} \frac{\cos\theta \sin^3\theta}{2\alpha^2} \exp\left(-\frac{\alpha}{\cos\theta}\right) \mathrm{d}\theta \right\} \qquad (9)$$

which is the same as that derived by Hiby and Pahl. Although the sum of N_w and N_g does not change when the effect of the variation in λ is considered, equations (7) and (8) are more exact than those due to Hiby and Pahl, hence the following treatment will be based on these equations.

2.2. *Nondiffusive reflection off the channel surfaces*

In the derivation of equations (5) and (6) it was assumed that molecules are diffusely reflected according to the cosine law off the channel surfaces. From the experimental results of Knudsen (1909), Gaede (1913), and Klose (1931) on the flow of gases through circular capillaries, Hiby and Pahl have shown that the momentum accommodation coefficient f for glass is 0.98 ± 0.01 and with this value of f their theory agrees with the experimental results to within 3%. However, for the flow experiments described in § 3, good agreement with the theory of Hiby and Pahl was only obtained for the flow of carbon dioxide through glass channels for values of α less than 0.2.

This difference in agreement between the two types of channels may be explained by the fact that the probability of molecules traversing long collision-free paths is much greater for a parallel-plate channel. Hence the contribution to the flow due to molecules leaving the channel surfaces at large values of θ is much greater for a parallel-plate channel than it is for a channel of circular cross section. But it is widely accepted that the probability of specular reflection increases as θ increases, hence it may be expected that the effect of specular reflection in parallel-plate channels is greater than that in channels of circular cross section. Figure 2 plots A, B, C and D
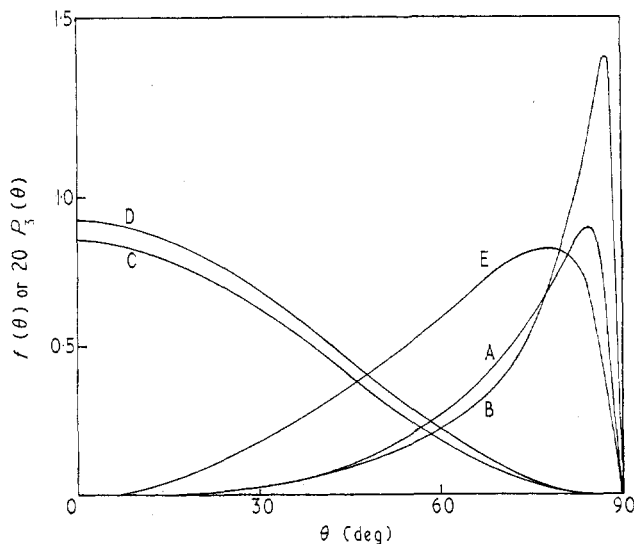

Figure 2. Variation of $f(\theta)$ with θ. A, B, for a parallel-plate channel $\alpha = 0.1$, $\alpha = 0.05$ respectively. C, D, for circular channel $\alpha = 0.1$, $\alpha = 0.05$ respectively. E shows the variation of $20\,P_3(\theta)$ with θ.

show the relative contributions to the N_w term of molecules leaving the surface at different values of θ, by assuming diffusive reflections, where the function $f(\theta)$ to be integrated in equation (7) is plotted against θ for $\alpha = 0 \cdot 1$ and $\alpha = 0 \cdot 05$, together with the corresponding values obtained from the equations derived by Present and Pollard for channels of circular cross section. The plots clearly demonstrate the effect of long collision-free paths in a parallel-plate channel and suggest how the flow may be further amended if the angular distribution of molecules leaving the surface at large values of θ does not follow the cosine law. In order to evaluate the effect on the the flow of the angular distribution of diffusive and specular reflection, consider the evidence from molecular beam experiments.

Numerous investigators have studied the scattering of molecular beams by solid surfaces, and have shown that the cosine law holds reasonably well for thermal-energy gas beams. Hinchen and Foley (1966) studied the scattering of thermal-energy gas beams by metal surfaces at low pressures. They observed diffusive scattering off contaminated surfaces but, at pressures below approximately 10^{-9} torr and after thorough surface cleaning, the scattering of the rare gases, nitrogen and atomic oxygen was quasispecular or lobular. Hurlbut (1959) studied the scattering of beams of nitrogen and of argon by various surfaces. He showed that for metal surfaces which had not been heated under a hard vacuum immediately before the experiment the scattering was diffusive but for unheated glass surfaces a small fraction of the incident beam was specularly reflected. An inspection of Hurlbut's polar diagrams of the reflected molecular intensity shows that for glass surfaces the probability of specular reflection increases with increasing angle of incidence, but that the lobe of specularly reflected molecules is not well defined.

However, from these results it is possible to estimate the angular distribution of molecules leaving the channel surfaces. It is assumed that molecules impinge on the surface with an isotropic spatial distribution; hence the number of molecules impinging on ds' from solid angle $d\omega'$ at θ' is

$$\frac{n \, ds'}{4\pi} \cos\theta' \, d\omega'.$$

If $A(\theta', \theta)$ is the beam signal amplitude measured by Hurlburt in arbitrary units at the reflected angle θ, then the probability that a molecule incident at θ' is reflected at angle θ is $P_1(\theta', \theta)$ where

$$P_1(\theta', \theta) = \frac{A(\theta', \theta)}{\int_0^{\pi/2} A(\theta', \theta) \sin\theta \, d\theta}.$$

Therefore the number of molecules leaving ds' in a solid angle $d\omega$ at θ is

$$\frac{n \, ds'}{4\pi} P_2(\theta) \, d\omega$$

where

$$P_2(\theta) = \int_0^{\pi/2} P_1(\theta', \theta) \sin\theta' \cos\theta' \, d\theta'.$$

In order to distinguish between the specular and diffusive terms, $P_2(\theta)$ is written in

the form $P_2(\theta) = a \cos\theta + P_3(\theta)$ where $P_3(0) = 0$ and where

$$a \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta + \int_0^{\pi/2} P_3(\theta) \sin\theta \, d\theta = \tfrac{1}{2}.$$

The values of a and $P_3(\theta)$ can be evaluated numerically (from the data given by Hurlbut) for both argon and nitrogen on glass surfaces. The results for argon are within 5% of those for nitrogen hence due to the experimental error in Hurlbut's work of approximately 10% both gases are represented by the same values of a and $P_3(\theta)$. The values of $P_3(\theta)$ are given in figure 2 and a was shown to be 0·927.

This angular distribution is now substituted for the cosine law in equation (5), and equation (7) then becomes

$$f_{\mathrm{w}}(\alpha) = \frac{2}{\pi} \int_0^{\pi/2} \left\{ a \cos\theta + P_3(\theta) \right\} \frac{\sin^3\theta}{\cos^2\theta} \exp\left(- \frac{\alpha}{\cos\theta} \right) d\theta. \qquad (10)$$

The values of $f_{\mathrm{w}}(\alpha)$ for different values of α can be calculated numerically for both argon and nitrogen flowing through glass channels and are shown in figure 3 together
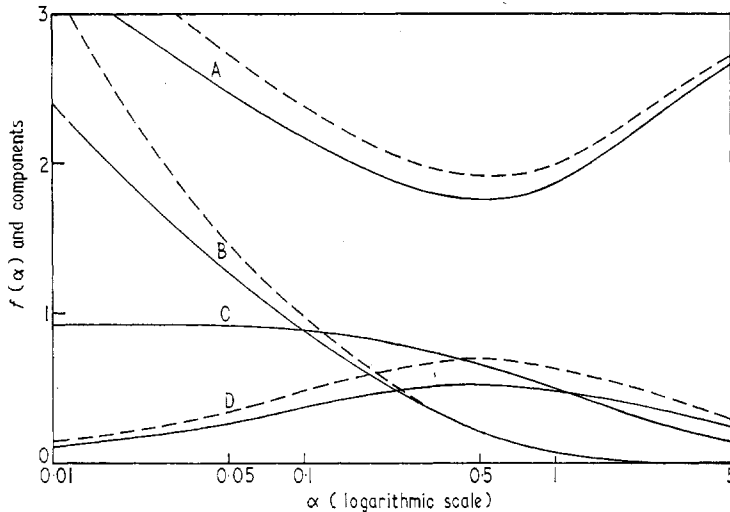

Figure 3. Variation of $f(\alpha)$ and components with α for: cosine law scattering (full lines), Hurlbut's scattering for nitrogen and argon (broken lines). A, $f(\alpha)$; B, $f_{\mathrm{w}}(\alpha)$; C, $f_{\mathrm{g}}(\alpha)$; D, $f_+(\alpha)$.

with the corresponding values calculated from equations (7) and (8). Unfortunately, similar experiments with the other gases were not carried out by Hurlbut, hence it is not possible to estimate quantitatively the effect of specular reflection on the lighter gases.

2.3. *Nonisotropic distribution of collision partners*

Up to now it has been assumed that after an intermolecular collision the collision partners move with isotropic distribution. But by assuming elastic collisions between the gas molecules Hiby and Pahl (1951) have shown that the emerging distribution of collision partners from a binary collision is anisotropic and may be represented by the diffusion function $W(\nu) = A - B \lg(1 - \cos\nu)$ where ν is the diffusion angle,

$A = 0.124$ and $B = 0.114$. Hiby and Pahl (1952) applied this diffusion function and derived an expression for the additional flow due to the anisotropic scattering of collision partners for both parallel-plate channels and channels of circular cross section. But their equations are only valid for values of α less than approximately 0.1 and where the reflection of molecules off the channel surfaces was according to the cosine law. In what follows, the method which they developed for anisotropic scattering in the gas phase for channels of circular cross section is now applied to a parallel-plate channel, for all values of α, and in addition account is taken of the modified angular distribution of molecules leaving the surfaces as summarized above.

In order to make the application of the diffusion function $W(\nu)$ possible, the 'subsidiary concept' developed by Hiby and Pahl (1952) is used, namely the molecular density of the channel element δl is assumed to comprise the two components n_2 and $n'(x)$, where $n(x) = n_2 + n'(x)$. The n_2 molecules have a constant density and possess an isotropic spatial distribution, whereas the n' molecules possess an anisotropic spatial distribution and their density varies from zero to $(dn/dl)\delta l$. It is also assumed that $n_2 \gg n'(x)$, hence collisions between n' molecules may be discarded on the account of their rarity and collisions between n_2 molecules do not contribute to the flow because of their isotropic distribution. With this classification, the kinetics of intermolecular collisions have been reduced to the problem of individual diffusion of n' molecules in isotropically moving n_2 molecules, thus the conditions for applying the diffusion function have been fulfilled.

Consider the element of volume $d\tau$ shown in figure 1 with an incident distribution E of n' molecules. The emerging distribution U is given by

$$U = \int_0^{4\pi} E \, W(\nu) \, d\omega'$$

where $d\omega' = \sin\theta' \, d\theta' \, d\phi'$. It is not possible to obtain a closed expression for U by analytic integration, which could then be used to calculate the additional flow due to anisotropic scattering N_+, therefore the following argument is used. In the case of an isotropic emerging distribution the flow would be given by the sum of equations (8) and (10). However, the actual process differs in that, for the anisotropic emerging distribution, each molecule traverses a 'supplementary jump' S between an intermolecular collision and its next collision. Hiby and Pahl have shown that the mean supplementary jump for a fixed incident direction θ and ϕ is

$$\bar{S}(\theta, \phi) = \frac{\displaystyle\int_0^{4\pi} S \, W(\nu) \, d\omega'}{\displaystyle\int_0^{4\pi} W(\nu) \, d\omega'}$$

and for a parallel-plate channel it may be shown that $S = \pm(\tfrac{1}{2}h - z) \tan\theta' \cos\phi'$. The incident distribution of n' molecules is the net flow through $d\tau$ at θ and ϕ given by the sum of dN_w and dN_g as used in the derivation of equations (8) and (10), hence

$$E(\theta, \phi) = \frac{\bar{c}}{4\pi} \frac{ds}{dx} \frac{dn}{dx} \sin^2\theta \, \cos^2\phi \left\{ \left(a - \frac{P_3(\theta)}{\cos\theta} \right) \left(r - \frac{r^2}{\lambda\cos\theta} \right) \exp\left(-\frac{r}{\lambda} \right) \right.$$
$$\left. + \lambda - \left(\lambda + r - \frac{r^2}{2\lambda} \right) \exp\left(-\frac{r}{\lambda} \right) \right\}.$$

Of this incident distribution a fraction of molecules $dx/\lambda \sin\theta \cos\phi$ will be involved in an intermolecular collision in $d\tau$. Therefore the number of molecules leaving $d\tau$ per unit time after colliding with another molecule, multiplied by the mean supplementary jump, is

$$\int_0^{4\pi} E(\theta, \phi) \frac{dx}{\lambda} \frac{\bar{S}(\theta, \phi)}{\sin\theta \cos\phi} d\omega.$$

But the total number of molecules in $d\tau$ is $n\, ds\, dx$, hence the mean velocity in the direction of flow of molecules leaving $d\tau$ after an intermolecular collision is

$$\bar{U} = \int_0^{4\pi} \frac{E(\theta, \phi)\, \bar{S}(\theta, \phi)}{\lambda n\, ds\, \sin\theta \cos\phi}\, d\omega$$

and the additional flow term becomes

$$N_+ = \int_{-b/2}^{b/2} \int_{-h/2}^{h/2} \bar{U} n\, dz\, dy. \tag{11}$$

Equation (11) has been evaluated numerically for both the cosine law molecular scattering and the scattering of argon and nitrogen off glass surfaces as observed by Hurlbut (1959). The values of $f_+(\alpha)$ where

$$N_+ = -\frac{\pi \bar{c} b h^2}{8} \frac{dn}{dx} f_+(\alpha)$$

are given in figure 3.

The total diffusive flow therefore becomes the sum of equations (8), (10) and (11). But in order to compare the theoretical values with the experimental results for values of α greater than $0\cdot1$, the viscous and slip flow terms, $f_v(\alpha)$ and $f_s(\alpha)$ respectively, as evaluated by Roberts (1969), are added to the diffusive term to give the general flow equation. Hence the general flow equation becomes .

$$Q = -\frac{h^2 b}{2l} \left(\frac{\pi M}{2RT}\right)^{1/2} f(\alpha) \Delta P \tag{12}$$

where $f(\alpha) = f_w(\alpha) + f_g(\alpha) + f_+(\alpha) + f_s(\alpha) + f_v(\alpha)$, Q is the mass rate of flow, ΔP is the differential pressure across the channel, M is the molecular weight of the gas, R is the gas constant, T is the temperature, l is the channel length and b is the channel breadth. The calculated values of $f(\alpha)$ and its components are given in figure 3 and are compared with previous work and the experimental results in figure 5.

3. Experimental procedure and results

The experiments were an extension of those described by Roberts (1969) in which the flow of rarefied gases and vapours between two parallel plates of separation between 7 and 40 μm was measured. However, in order to measure the flow rates when the mean pressure in the channel is much greater than the differential pressure across the channel, a new system had to be designed in which any one flow rate is measured at constant pressure by continuously changing the volume of the high pressure side of the channel at the appropriate rate by means of a small piston. With such a system the same procedure could be used for both gases and vapours, since the effect of absorption and desorption of water vapour off the surfaces of the apparatus could be neglected.

The flow rates were measured by observing the speed of the piston shown in figure 4, when the pressure of the gas between the piston and the channel was constant. Hence if P_f and P_b are the pressures in front of and behind the channel respectively the mass rate of flow Q is given by

$$\underset{\sim}{Q} = \frac{P_f M}{RT} \frac{dV}{dt}$$

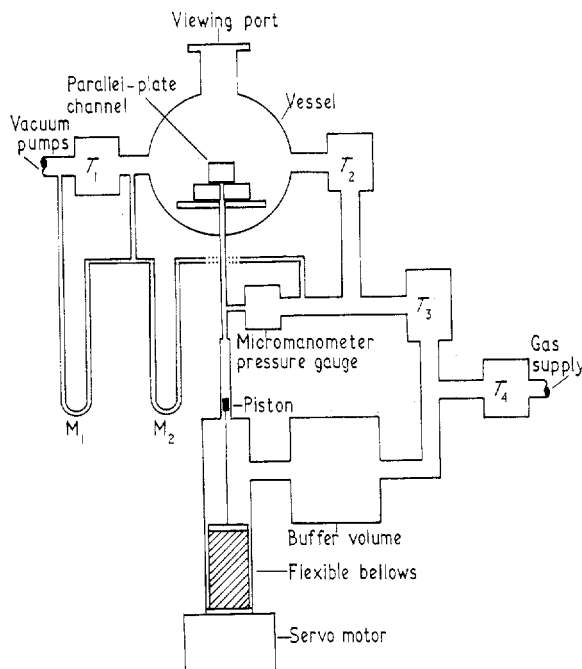where dV/dt is the volume rate of flow.

Figure 4. The apparatus.

The vacuum system was made from stainless steel with copper gasket seals, except for the silicon 704 oil manometers which were made of glass and the PTFE piston which was lubricated with silicon oil 705. Each individual component had a leak rate of less than 10^{-9} torr l s^{-1}. The diameter of the piston was 1·9 cm and the total length through which it could move was approximately 10 cm. In order to keep the pressure P_b constant for the duration of an experiment the parallel-plate channel was placed in a vessel of volume 12 litres, thus the fractional change in P_b during an experiment was at all times less than 0·2%. The pressure P_f was kept constant by means of the micromanometer differential pressure gauge shown in figure 4, which was electrically coupled to the servo-motor driving the piston. The gauge was a Melinex-diaphragm differential-capacitance gauge and the coupling with the servo-motor was arranged so that the pressure P_f was kept within $\pm 0·005$ torr to the reference pressure trapped between the gauge and values T_2 and T_3. Hence the fractional variation in P_f during an experiment was always less than 1%. This was the best arrangement possible since an exact calibration of the gauge was not necessary and, owing to the theoretically zero differential pressure across the gauge, the

4A

leakages due to the porosity of the diaphragm could be neglected. In order to reduce the leakage past the piston, the differential pressure across the piston was kept as small as possible by means of the 2·5 litre buffer volume. Therefore, by closing values T_3 and T_4 at the beginning of each experiment, the gas trapped behind the piston was at the pressure P_f. The largest differential pressure across the piston was approximately 0·5 torr and at this pressure the estimated leak rate was less than 10^{-9} torr l s^{-1}.

The pressures P_f and P_b were measured with the manometers M_1 and M_2 and by employing a cathetometer to observe the difference in height of the oil levels an accuracy of ± 0.005 torr was obtained which, for the lowest values of P_f and P_b used, gave errors of less than 1%. The main source of error in all the experiments was due to the measurement of h. For channels where h was greater than 6 μm, the error in h^2—which is the relevant term—was less than 2%, but for the smallest value of h used the error in h^2 was approximately 4%.

The channels were prepared in the manner described in the previous paper and the separation h measured by observing Edser–Butler interference fringes by means of the viewing port shown in figure 4. Before any flow experiments were carried out, and between experiments with different gases and vapours, the whole system was evacuated to a pressure of less than 5×10^{-6} torr and kept at that pressure until the effects of desorption were less than 10^{-8} torr l s^{-1}. The procedure for all the experiments was such that the gas or vapour used was allowed to flow under equilibrium conditions through the channel for at least 20 minutes in the case of the gases and 2 hours in the case of water vapour, before commencing with the actual flow experiment. Hence both thermal and adsorption–desorption equilibrium could be assumed. The temperature of the system during an experiment was kept within ± 0.5 to 20° C for all the gases and to within ± 0.1° C for water vapour.

One set of results was obtained for a silver channel, which had been prepared by evaporating silver on to the optically flat glass blocks, with the upper block being only partially silvered so that the separation h could be determined.
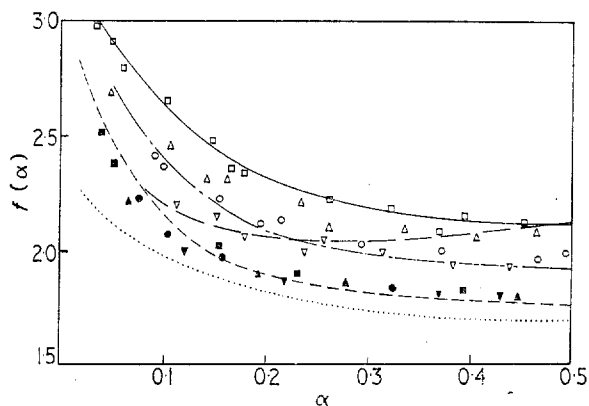

Figure 5. Variation of $f(\alpha)$ with α. $\triangle$ hydrogen on glass, $\square$ helium on glass, $\odot$ nitrogen and argon on glass, $\triangledown$ carbon dioxide, krypton and water vapour on glass, $\blacktriangle$ hydrogen on silver, $\blacksquare$ helium on silver, $\bullet$ nitrogen and argon on silver, $\blacktriangledown$ carbon dioxide, krypton and water vapour on silver. Hiby and Pahl equation (3, 18) —— ——; mean of the results for helium ———; present theory with cosine law scattering – – –; present theory with Hurlbut scattering for nitrogen and argon —— - ——; mean of Dong's results ······.

In order to compare the experiments carried out at different values of h and differential pressure ΔP, the results are presented on a graph of $f(\alpha)$ plotted against α shown in figure 5, where $f(\alpha)$ is as shown in equation (12). The variation of Q with ΔP or h is not shown since, to within the experimental error, it follows that given in equation (12), where $f(\alpha)$ is a function of α and of the solid–gas combination only. Figure 5 shows the results for all the gases and vapours used for both glass and silver channels, but for clarity some of the gases and vapours which gave similar results have been grouped together. From figure 5 it is readily observed that in the case of the glass channels the values of $f(\alpha)$ increase inversley with the constant $\beta = \lambda P$, as defined by Roberts (1969). The values of β at a temperature of $20°$C are as follows: hydrogen 12·21, helium 19·03, nitrogen 6·440, argon 6·893, carbon dioxide 4·318, krypton 5·271, and water vapour 4·530. The values of $f(\alpha)$ for the silver channel are consistently lower than those for the glass channels and the variation of $f(\alpha)$ with β is not observed.

The results for carbon dioxide, krypton and water vapour agree within 3% with Gaede's results for the flow of carbon dioxide between two parallel glass plates and with equation 3·18 due to Hiby and Pahl (1952). However Hiby and Pahl's equations do not explain the variation of $f(\alpha)$ with the gas or vapour used nor do they explain the lower values of $f(\alpha)$ obtained with the silver channel.

4. Discussion

The good agreement shown in figure 5 between the theoretical evaluation and the corresponding experimental results clearly demonstrates that the rate of flow depends on the nature of molecular reflections off the channel surfaces. Unfortunately Hurlbut (1959) did not extend his molecular beam experiments to cover a wide range of gases but Hinchen and Foley (1966) and Saltsburg *et al.* (1967) have shown that the degree of specular reflection generally decreases as the molecular weight increases and is also relatively higher for the inert gases. In the case of water vapour, although its molecular weight is lower than that of most of the gases used, the experiments indicate that the degree of specular reflection is comparable to that of carbon dioxide and krypton. This may be explained by the higher affinity of water vapour towards glass surfaces. However, it must be appreciated that the strong dependence of the flow rate on the angular distribution of specular reflection is not observed with channels of circular cross section. Hiby and Pahl (1952) have shown that to within approximately 2% the results of Knudsen (1909) for oxygen, carbon dioxide and hydrogen, and the results of Gaede (1913) for hydrogen and nitrogen, all flowing through glass circular capillaries, may be represented by their equation (2,1); hence the difference between the gases, apparent with the parallel-plate channels, was not observed.

Dong (1956) carried out experiments on the flow of hydrogen, helium, air, carbon dioxide and freon-12 between two parallel plates of separation 0·324 cm. The randomness of his results indicates an experimental error of greater than 10% for values of α less than 0·2; however, the mean of all his results is given in figure 5 and is approximately 5% lower than that calculated in this work.

Roberts (1969) using similar channels to those used in the present work does not report any variation between the gases and vapours due to specular reflection; however, these earlier experiments were carried out for values of α greater than 0·5 where the effects observed in the present experiments were within the experimental error of the previous work.

The results of the experiments carried out with the silver channel indicate that the surface scattering for all the gases used can be assumed to be wholly diffusive, which agrees well with the molecular beam experiments mentioned earlier.

Acknowledgments

References

DONG, W., 1956, *Univ. Calif. Rep.*, UCRL—3353.
FRYER, G. M., 1966, *Proc. R. Soc.*, A, **293**, 329–41.
GAEDE, W., 1913, *Ann. Phys. Lpz.*, **41**, 289–336.
HIBY, J. W., and PAHL, M., 1951, *Z. Phys.*, **129**, 517–29.
—— 1952, *Z. Naturf.*, **7a**, 533–53.
HINCHEN, J. J., and FOLEY, W. M., 1966, *Rarefied gas dynamics*, Ed. J. H. de Leeuw (New York: Academic Press), pp. 505–17.
HURLBUT, F. C., 1959, *Recent research in molecular beams*, Ed. I. Estermann (New York: Academic Press), pp. 145–56.
KLOSE, W., 1931, *Ann. Phys.*, **11**, 73–93.
KNUDSEN, M., 1909, *Ann. Phys. Lpz.*, **28**, 75–130.
PRESENT, W. G., and POLLARD, R. D., 1948, *Phys. Rev.*, **73**, 762–74.
ROBERTS, G. T., 1969, *J. Phys.* A: *Gen. Phys.*, **2**, 685–96.
SALTSBURG, H., SMITH, J. N., and PALMER, R. L., 1967, *Rarefied gas dynamics*, Ed. C. L. Brundin (New York: Academic Press), pp. 223–35.
VON SMOLUCHOWSKI, M., 1910, *Ann. Phys. Lpz.*, **33**, 1559–70.
WEBER, S., 1954, *K. danske Vidensk. Selsk., Math.-fys. Meddr.*, **28**, 1–138.